

Notes on Properties of Exponents

The Product Rule: $a^m \cdot a^n = a^{m+n}$ (1) $x^6 \cdot x^1 = x^{6+1} = \boxed{x^7}$

add exponents

The Quotient Rule $\frac{a^m}{a^n} = a^{m-n}$ (2) $\frac{3^6}{3^4} = \boxed{9}$ (3) $\frac{y^{15}}{y^8} = \boxed{y^7}$ (4) $\frac{w^4}{w^4} = \boxed{1}$
 $3^{6-4} = 3^2 = 9$ $y^{15-8} = y^7$ $w^{4-4} = w^0 = 1$

subtract exponents

The Power Rule $(a^m)^n = a^{mn}$ (5) $(2^3)^2 = 2^3 \cdot 2^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \boxed{2^6 \text{ or } 64}$

multiply exponents

(6) $(m^5)^4 = m^{5 \cdot 4} = \boxed{m^{20}}$

Product to a Power $(ab)^n = a^n b^n$ (7) $(7x)^2 = 7^2 x^2 = \boxed{49x^2}$

(8) $(x^2 y^5 z)^3 = \boxed{x^6 y^{15} z^3}$

Quotient to a Power $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (9) $\left(\frac{a^3}{b^2}\right)^4 = \frac{a^{3 \cdot 4}}{b^{2 \cdot 4}} = \boxed{\frac{a^{12}}{b^8}}$

(10) $(3ab^4)(2a^2b)^3$
 $(3ab^4)(2^3 a^6 b^3)$

$24 a^{1+6} b^{4+3}$

$\boxed{24 a^7 b^7}$

(11) $\left(\frac{x^2 y^4}{2xz}\right)^3$

$\left(\frac{x^{2 \cdot 3} y^{4 \cdot 3}}{2z}\right)^3$

$\left(\frac{x^6 y^{12}}{2z^1}\right)^3 = \boxed{\frac{x^3 y^{12}}{8 z^3}}$

***Other Properties

$a^1 = a$

$a^0 = 1$

Negative Exponents $a^{-n} = \frac{1}{a^n}$ (12) $3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

$$a^{-n} \cdot a^n = a^0 = 1 \quad \frac{1}{a^n} \cdot \frac{a^n}{1} = \frac{a^n}{a^n} = a^0 = 1$$

Reciprocal a^{-n} and a^n are reciprocals

$$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n} \quad (13) \quad \frac{2^{-2}}{5^{-2}} = \frac{5^2}{2^2} = \boxed{\frac{25}{4}}$$

If we move a factor across the fraction bar the sign of the exponent will change.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad (14) \quad \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \boxed{\frac{16}{9}}$$

Any base to an exponent is equal to the reciprocal of the base if raised to the opposite exponent.

Typically an expression is not considered simplified if it contains negative exponents

(1) $2^{-5} \cdot 2^2 = 2^{-5+2} = 2^{-3}$

$$\frac{1}{2^3} = \boxed{\frac{1}{8}}$$

(2) $\frac{x^{-12}}{x^{-8}} = X^{-12+8}$

$$= \frac{x^{-4}}{1} = \boxed{\frac{1}{x^4}}$$

(3) $(6m^{-4}n^6)(2m^{-2}n)$

$$12m^{-4-2}n^{6+1} = \boxed{\frac{12n^7}{m^6}}$$

(4) $(a^{-9})^{-4}$

$$a^{-9 \cdot -4} = \boxed{a^{36}}$$

(5) $\frac{-15x^{-1}y^4z^{-3}}{45x^4yz^2}$

$$\frac{-1x^{-1+4}y^{4-1}z^{-3-2}}{3} = \frac{-1x^3y^3z^{-5}}{3}$$

$$\frac{-1x^3y^3}{3z^5} = \boxed{\frac{-1x^3y^3}{3z^5}}$$

(6) $\left(\frac{x^{-4}}{y^{-3}}\right)^2 = \frac{x^{-8}}{y^{-6}}$

$$= \boxed{\frac{y^6}{x^8}}$$

(7) $\frac{(a^2)^3(a^{-2}b)^7}{(a^{-1}b)^2}$

$$\frac{a^6 a^{-14} b^7}{a^{-2} b^2}$$

$$\frac{a^{-8} b^7}{a^{-2} b^2}$$

$$= \frac{a^{-8+2} b^{7-2}}{1} = \boxed{\frac{b^5}{a^6}}$$